Risk-Averse Airline Revenue Management with Coherent Measures of Risk

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Proposal

Outline

Revenue Management

- Airline Industry
- Overbooking
- No-shows

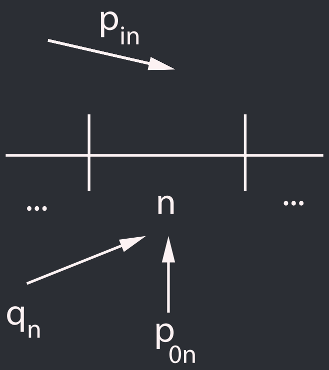
Risk Neutral Model

We need to divide the time horizon into discrete periods.

Risk Neutral Model

In each period, there can be three events:

- p_{ni} is the probability of a request for a seat in class i.
- q_n is the probability of cancellation.
- p_{0n} is the probability of a null event.



Risk Neutral Model

If a request is accepted for class i at stage n, $r_{in} \geq 0$ is the amount of earned revenue.

We have

$$\sum_{i=1}^{m} p_{ni} + q_n + p_{0n} = 1 \quad \textit{for all } n \geq 1.$$

Risk Neutral Model

We also have no-shows. Each customer has a probability β of no-show at the departure time. x is the state.

Definition

$Y(x)$: number of customers show up at stage 0.

$Y(x) \sim Binomial(x, 1-\beta)$. Let $Y(x) = y$ at the time of departure.

Definition

$\pi(y)$: overbooking penalty function.

Risk Neutral Model

Definition

$U_n(x)$ is the maximum expected revenue over periods n to 0.

At period 0,

$$U_0(x) = E[-\pi(Y(x))], \quad 0 \leq x < N$$

We have recursive function as

$$U_n(x) = \sum_{i=1}^{m} p_{in} max\{r_{in} + U_{n-1}(x+1), U_{n-1}(x)\}$$
$$+ q_n U_{n-1}(x-1) + p_{0n} U_{n-1}(x)$$

Risk Neutral Model

Booking Limit Policy

In order to define a control limit policy we need to show that

$U_n(x) - U_n(x+1)$ is nondecreasing in $x = 0, 1, \ldots, N - n - 1$.

We use the following lemmas.

Lemma 1 Let $f(y)$, $y \geq 0$, be a nondecreasing convex function. For each non-negative integer x, let $Y(x)$ be a binomial (x, γ) random variable$(0 < \gamma < 1)$ and let $h(x) = E[f(Y(x))]$. Then $h(x)$ is nondecreasing convex in $x \in \{0, 1, \ldots\}$.

Lemma 1 has been proved by Shaked and Shanthikumar (1994).

Risk Neutral Model

Booking Limit Policy

Lemma 2 Let $p_{0n} = \bar{q}_n - q_n$ where $\bar{q}_n$ stands for 1 - $\sum_{i=1}^{m} p_{in}$. Let $H(x) = q_n U_{n-1}(x-1) + (\bar{q} - q_n) U_{n-1}(x)$. If $U_n(x)$ is a nonincreasing concave function in x then $H(x)$ is a nonincresing concave function in x.

Lemma 3 Let $r_{in} \geq 0$ is fixed and $g(x) = max\{r_{in} + U_{n-1}(x+1), U_{n-1}(x)\}$. If $U_{n-1}(x)$ is concave and nonincresing, then $g(x)$ is concave and nonincreasing.

Lemma 2 and Lemma 3 have been proved by Lippman and Stidham (1977).

Risk Neutral Model

Booking Limit Policy

$U_n(x) - U_n(x+1)$ can be seen as opportunity cost of accepting a request at stage $n+1$. Let b_{in} be the booking limit at stage n and class i. It is defined as,

$$b_{in} := min\{x : U_{n-1}(x) - U_{n-1}(x+1) > r_{in}\}$$

Risk Neutral Model

Booking Limit Policy

After all, our booking limit policy is the following,

accept a request for fare class i in state x at stage n
$\Leftrightarrow 0 \leq x < b_{in}$.

- We will show whether a control limit policy exists for risk-averse model with coherent measures of risk.

Risk Averse Model

Let $\rho(\cdot)$ be a coherent risk measure. Using first order mean semi-deviation representation as the coherent risk measure we have following dynamic programming equations.

$$V_0(x) = \mathbb{E}[-\pi(Y(x))] - \kappa\mathbb{E}\Big[\mathbb{E}[-\pi(Y(x))] + \pi(Y(x))\Big]_+$$

$$V_n(x) = \mu_n - \kappa\Big[\sum_{i=1}^{m} p_{ni}\big[\mu_n - max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\}\big]_+$$

$$+q_n\big[\mu_n - V_{n-1}(x-1)\big]_+ + p_{0n}\big[\mu_n - V_{n-1}(x)\big]_+\Big]$$

Risk Averse Model

Where;

- $\kappa \in [0, 1]$
- $\mu_n = \sum_{i=1}^{m} p_{ni} max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\}$
  $+ q_n V_{n-1}(x-1) + p_{0n} V_{n-1}(x)$

Risk Averse Model

Dual Representation

It is very hard to show that $V_n(x)$ is concave and nonincreasing. So, we will use the dual representation.

Let Y be a random reward. Then, dual representation of $\rho(\cdot)$ is

$$\rho(Y) = \min_{\eta \in A} \mathbb{E}_\eta[Y]$$

(Artzner et al. (1999))

Risk Averse Model

Dual Representation

Then, dual representation of mean semi-deviation,

$$V_0(x) = \min_{\eta \in A_0} \mathbb{E}_\eta[-\pi(Y(x))]$$

$$V_n(x) = \min_{\eta \in A_n} \sum_{i=1}^{m} \eta_i max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\}$$

$$+\eta_{m+1}V_{n-1}(x-1) + \eta_{m+2}V_{n-1}(x)$$

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References II

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