Risk-Averse Airline Revenue Management with Coherent Measures of Risk

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Abstract

Revenue management aims to sell the right product to the right customer at the right time. Airline industry has a leading role in revenue management. Overbooking is a profitable strategy which is widely used by airline firms. In this setting, an airline firm needs to make critical decisions such as to accept a customer request for a seat in a specific time or to overbook a request in order to manage its revenue. This problem has been modelled using Markov Decision Processes. In the literature, control limit policy for risk neutral models is a well-studied subject. In this study, our aim is to show whether a control limit policy exists for single-leg risk-averse model with overbooking, cancellation and no-shows under coherent measures of risk.

Keywords: Revenue Management, Risk-Averse Markov Decision Processes, Control Limit

1. Introduction

Revenue management is a branch which tries to optimize price and availibility of products to sustain maximum revenue and growth. Airline industry is the area where revenue management is widely used and developed. In the airline industry, seat is the resource and product which can be considered as a perishable good. Stochastic and seasonable demand pattern and high costs make the decisions harder in the industry and basing decisions on a rationale is required. Capacity allocation and overbooking are two main strategies used for revenue management in airline industry.

2. Problem Definition

Our specific problem is determining booking limits which can be used for decision making at each stage. In order to solve the problem, one way is to model the process as a Markov Decision Process which can be solved using Dynamic Programming. In order to model the process, we need to divide the time period which starts with the opening of seat requests for a particular leg of flight and ends with departure into small periods. In this setting, our state will be the current number of reserved seats. Moreover, according to our assumption, within a discrete-time period only one of the following events can happen: an arrival of a new passenger, a cancellation out of existing customers or a null event which basicly does not affect state. Overbooking is allowed in our problem.

Using this setting, the modelling is done by Selin Özbek who is a previous IE 491 student of Özlem Çavuş and numerical results are received. A control limit is the number that can be used in decision making in a time period after an event occurs. Control limits need to be calculated for all periods in order to construct a control limit policy. For our problem, we expect to find a control limit policy in the following fashion: for each period, accept an arriving customer of a fare class if the current state is less than the control limit value found for that period and class, otherwise reject the customer. In the literature, control limit policy for risk neutral case is a well-studied subject. In this project, our aim is to show whether a control limit policy exists for single-leg risk-averse model with coherent measures of risk.

3. Literature Review

Subramanian et al. (1999) analyze a Markov decision process model for airline seat allocation. They have a single-leg flight with multiple fare classes. Their models allow cancellation, no-shows and overbooking. They show that an optimal policy is characterized by state and time dependent booking limits. They have two models. Their first model allows cancellation and no-show probabilities to be time dependent and class independent. In their second model, they allow cancellation and no-show probabilities to be class dependent. Their models are risk neutral. Furthermore, they show with numerical example that their model have 9% revenue gain.

Aydın et al. (2013) develop both dynamic and static single-leg overbooking models. Their study aims to determine booking limits and capacity allocation among fare classes. In the static case, they have two models. Their first static model considers a greedy policy that accepts a request for any class as long as booking limit is not exceeded. Their booking limit does not depent on the probability distribution of demand. Their second model is about finding overbooking limit and allocating the overbooking-added capacity among classes.The second static model is a structure that is not preferred among practitioners. In their dynamic case, they propose a dynamic programming model. Their model is based on two streams of events, the first one is arrival of booking requests and the second one is the cancellations.

4. Risk Neutral Model

There are m fare classes and N decision periods. In a period $n = N, N-1, ..., 1, 0$; p_{ni} is the probability of a request for a seat in class i, where $i = 1, 2, ..., m$. Let x denote the state i.e. the number of reserved seats. q_n is the probability of cancellation in period n. p_{0n} is the probability of a null event in period n. Let C denote the capacity of plane. If a request is accepted for class i at stage n, $r_{in} \geq 0$ is the amount of earned revenue.

We have

$$\sum_{i=1}^{m} p_{ni} + q_n + p_{0n} = 1 \quad for\ all\ n \geq 1. \tag{1}$$

Period 0 stands for the stage when the plane takes off. Each customer has a probability β of no-show at the departure time. Let $Y(x)$ denote the number of customers show up at stage 0. $Y(x)$ has binomial$(x, 1-\beta)$ distribution. Let $Y(x) = y$ at the time of departure. We also have overbooking penalty function $\pi(y)$ which is a nondecreasing convex function of y. For $Y(x) \leq C$, $\pi(y) = 0$.

We model the problem as Markov Decision Process. Our objective is to maximize expected total revenue from period N to period 0. Let $U_n(x)$ denote the maximum expected revenue over periods n to 0. We have the recursive function as

$$U_n(x) = \sum_{i=1}^{m} p_{in} max\{r_{in} + U_{n-1}(x+1), U_{n-1}(x)\} + q_n U_{n-1}(x-1) + p_{0n} U_{n-1}(x),$$
$$0 \leq N-n, \quad n \geq 1 \quad (2)$$

And we have base case at period 0 as

$$U_0(x) = E[-\pi(Y(x))], \quad 0 \leq x < N \quad (3)$$

In order to define a control limit policy we need to show that $U_n(x) - U_n(x+1)$ is nondecreasing in $x = 0, 1, ..., N-n-1$. The following lemma is from Example 6.A.2 in Shaked and Shanthikumar, 1994.

Lemma 1 Let $f(y), y \geq 0$, be a nondecreasing convex function. For each nonnegative integer x, let $Y(x)$ be a binomial (x, γ) random variable$(0 < \gamma < 1)$ and let $h(x) = E[f(Y(x))]$. Then $h(x)$ is nondecreasing convex in $x \in \{0, 1, ...\}$.

Note that, $\pi(Y(x))$ is a nondecreasing convex function. Then $-\pi(Y(x))$ is a concave and nonincreasing function. Using Equation 3 and Lemma 1, $U_0(x)$ is a concave and nonincreasing function.

Lemma 2 Let $p_{0n} = \bar{q}_n - q_n$ where $\bar{q}_n$ stands for 1 - $\sum_{i=1}^{m} p_{in}$ from Equation 1. Let $H(x) = q_n U_{n-1}(x-1) + (\bar{q} - q_n) U_{n-1}(x)$. If $U_n(x)$ is a nonincreasing concave function in x then $H(x)$ is a nonincresing concave function in x.

Lemma 3 Let $r_{in} \geq 0$ is fixed and $g(x) = max\{r_{in} + U_{n-1}(x+1), U_{n-1}(x)\}$. If $U_{n-1}(x)$ is concave and nonincresing, then $g(x)$ is concave and nonincreasing.

Lemma 2 and Lemma 3 have been proved by Lippman and Stidham.

Lemma 4 $U_n(x)$ is a concave and nonincreasing function.

Proof: $U_0(x)$ is concave and nonincreasing. Assume that $U_{n-1}(x)$ is concave and nonincreasing. Using Lemma 2, the second and third terms of Equation 2 compose a nonincreasing concave function. Moreover using Lemma 3, the first part of Equation 2 is a nonincreasing concave function. Since sum of concave functions is concave, $U_n(x)$ is a concave and nonincreasing function.

Lemma 5 $U_n(x) - U_n(x+1)$ is nondecreasing.

Proof: Let $f(x) = U_n(x) - U_n(x+1)$. Then, $f(x) - f(x+1) = U_n(x) - U_n(x+1) - [U_n(x+1) - U_n(x+2)] \leq 0$ because of the concavity of $U_n(x)$. So, $f(x)$ is a nondecresing function.

$U_n(x) - U_n(x+1)$ can be seen as opportunity cost of accepting a request at stage $n+1$. Let b_{in} be the booking limit at stage n and class i. It is defined as,

$$b_{in} := min\{x : U_{n-1}(x) - U_{n-1}(x+1) > r_{in}\} \tag{4}$$

Using the booking limit, we have an optimal policy as,

accept a request for fare class i in state x at stage $n \Leftrightarrow 0 \leq x < b_{in}$.

5. Risk-Averse Model

We use mean semi-deviation representation of a random reward:

$$V_0(x) = \mathbb{E}[-\pi(Y(x))] - \kappa\,\mathbb{E}\left[\mathbb{E}[-\pi(Y(x))] + \pi(Y(x))\right]_+, \qquad \kappa \in [0,1] \tag{5}$$

$$\begin{aligned} V_n(x) = \mu_n - \kappa\Bigg[\sum_{i=1}^{m} p_{ni}\big[\mu_n - max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\}\big]_+ \\ + q_n\big[\mu_n - V_{n-1}(x-1)\big]_+ + p_{0n}\big[\mu_n - V_{n-1}(x)\big]_+\Bigg], \\ \kappa \in [0,1] \end{aligned} \tag{6}$$

where

$$\mu_n = \sum_{i=1}^{m} p_{ni} max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\} + q_n V_{n-1}(x-1) + p_{0n} V_{n-1}(x) \tag{7}$$

To examine the booking limit policy, we need to show that $V_n(x)$ is a concave and nonincreasing function. We will use dual representation for examining concavity. Let g(.) be a coherent risk measure and Y be a random reward. We can show the dual representation of g(.) as

$$g(Y) = \min_{\eta \in A} \mathbb{E}_\eta[Y] \tag{8}$$

where A is a closed and convex set. Using Equation 8 we have the dual representations of mean semi deviation,

$$V_0(x) = \min_{\eta \in A_0} \mathbb{E}_\eta[-\pi(Y(x))] \tag{9}$$

$$V_n(x) = \min_{\eta \in A_n} \sum_{i=1}^{m} \eta_i max\{V_{n-1}(x+1) + r_{in}, V_{n-1}(x)\} + \eta_{m+1} V_{n-1}(x-1) + \eta_{m+2} V_{n-1}(x) \tag{10}$$

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